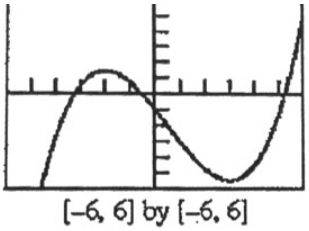
AP Calculus AB

PRACTICE MIDTERM EXAM

Read each choice carefully and find the best answer. Your midterm exam will be made up of 45 of these questions. I reserve the right to change numbers and answers on the actual exam.

NO CALCULATOR

1. Find the interval or intervals on which the function whose graph is shown is increasing:



A) $(-\infty,-3] \cup [5,\infty)$ B) $[-2,3]$ C) $(-\infty,-2] \cup [3,\infty)$ D) $[-3,5]$

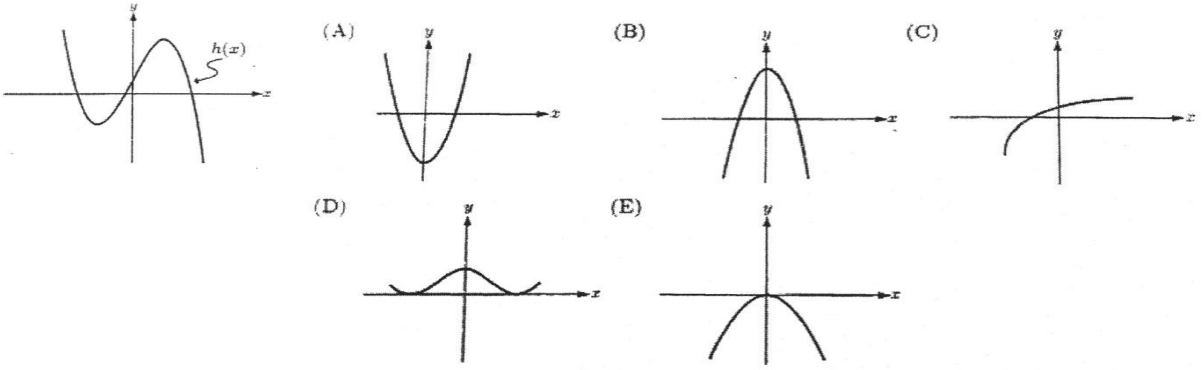
2. Does the graph of $f(x)=\dfrac{6x^2+2x-4}{2x^2+3x+2}$ have a horizontal asymptote? If so what is it?

a. yes, y=3 b. yes, y=-3 c. yes, y=2 d. yes, y=4 e. no

3. The position of a particle moving along a coordinate line is $s=\sqrt{3+6t}$, with s in meters in t in seconds. Find the particle's velocity at t=1sec.

a. 1/6 m/s b. -1/3 m/s c. 2 m/s d. 1 m/s e. -1/6 m/s

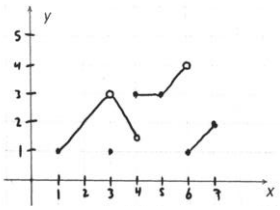
4. The graph of the original function is shown. Which of the following could be the graph of the derivative function?



5. $\lim\limits_{x\to -3}\dfrac{x^2+3x}{\sqrt{x^2+6x+9}}$

a. -3 b. -1 c. 1 d. 3 e. DNE

6. The graph of a function f whose domain is the closed interval [1,7] is shown below. Which of the following statements about f(x) is TRUE?



a. $\lim_{x\to 3} f(x) = 1$ b. $\lim_{x\to 4} f(x) = 3$ c. f(x) is continuous at x=3

d. f(x) is continuous at x=5 e. $\lim_{x\to 6} f(x) = f(6)$

7. $\lim_{x\to 3} \dfrac{x^2-9}{x^3-27}$

a. -2/9 b. 2/9 c. 0 d. 3 e. DNE

8. $\lim_{x\to 3^-} \dfrac{|x-3|}{3-x}$

a. $-\infty$ b. -1 c. 0 d. 1 e. ∞

9. $\lim_{x\to b} \dfrac{b-x}{\sqrt{x}-\sqrt{b}}$

a. $-2\sqrt{b}$ b. $-\sqrt{b}$ c. $2b$ d. $\sqrt{b}$ e. $2\sqrt{b}$

10. $\lim_{x\to 3} \dfrac{(3-x)^2}{(x-3)}$

a. 0 b. -2 c. 1 d. -1 e. DNE

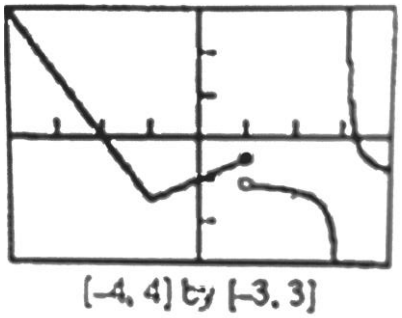
11. $\lim_{x\to 7} \dfrac{x^2+2x-63}{x-7}$

a. DNE b. 16 c. 2 d. 0 e. ∞

12. Find the equation of the line tangent to the graph of $x^2+y^2=12$, at the point $(\sqrt{3},3)$.

a. $y=-\dfrac{\sqrt{3}}{3}x+3$ b. $y=\dfrac{\sqrt{3}}{3}x+2$ c. $y=-\dfrac{\sqrt{3}}{3}x+4$ d. $y=\sqrt{3}x+\dfrac{\sqrt{3}}{3}$ e. none of these

13. The graph of $y=f(x)$ is shown below. At what values of x does f(x) appear to be nondifferentiable?



a. x=1, x=3 b. x=-1, x=1, x=3 c. x=-1, x=1 d. x=-1, x=1, x=-3 e. none of these

14. Find the value or values of c that satisfy the mean value theorem for the function $f(x) = x + \frac{48}{x}$, on the interval [3,16].

a. $0, 4\sqrt{3}$ b. 3,16 c. $4\sqrt{3}$ d. $-4\sqrt{3}, 4\sqrt{3}$ e. none of these

15. Find the relative/local extrema for $h(x) = \frac{x-1}{x^2+5x+10}$.

a. rel min at x=-4, rel max at x=5
b. rel min at x=-3, rel max at x=5
c. rel min at x=-3, no rel max
d. no rel extrema

16. On the interval (1,2), the curve $y = x^3 - 6x^2 + 9x + 1$ is,

a. increasing and CCU b. increasing and CCD c. decreasing and CCU d. decreasing and CCD e. horizontal

17. Find the intervals where the graph of the function is concave up if $y = x^3 - 3x^2 - 9x + 3$

a. $(1, \infty)$ b. $(-\infty, 1)(1, \infty)$ c. $(-\infty, 1)$ d. $(-\infty, \infty)$ e. none

18. A sphere is increasing in volume at the rate of $3\pi cm^3/s$. At what rate is its radius changing when the radius is ½ cm? $(V = \frac{4}{3}\pi r^3)$

a. $\pi cm/s$ b. 3cm/s c. 2cm/s d. 1cm/s e. ½ cm/s

19. Find a value of a so that the function $f(x) = \begin{cases} 6x - 5, x < 4 \\ ax^2, x \geq 4 \end{cases}$ is continuous.

a. 6/4 b. a=16 c. a=19/16 d. a=19/4 e. 16/19

20. $\lim_{x \to -\infty} \frac{4x^3 + 3x^2}{x - 6x^2}$

a. ∞ b. $-\frac{1}{2}$ c. $-\infty$ d. 0 e. 4

21. Let $\lim_{x \to 10} f(x) = 1$ and $\lim_{x \to 10} g(x) = 5$. Find $\lim_{x \to 10} [f(x) + g(x)]^2$.

a. 26 b. 36 c. 6 d. -4 e. 12

X	f(x)	g(x)	f'(x)	g'(x)
3	-3	6	-5	1
4	0	3	-3	9
5	3	-2	4	5

22. Using the table above find $h'(5)$, if $h(x) = f(x)g(x)$

a. 2 b. 7 c. 14 d. 20 e. 26

23. Using values from the table above find $h'(4)$, if $h(x) = f(g(x))$

a. -45 b. -27 c. -15 d. 0 e. 25

24. Find the derivative of $f(x)=\frac{6+x}{1-x}$, for x=2.

a. -7 b. $\frac{1}{7}$ c. 7 d. 10 e. $-\frac{1}{7}$

25. Find $\frac{dy}{dx}$ by implicit differentiation for $\cos(xy)+x^5=y^5$

a. $\frac{5x^4-y\sin xy}{5y^4+x\sin xy}$ b. $\frac{5x^4+y\sin xy}{5y^4}$ c. none of these d. $\frac{5x^4+y\sin xy}{5y^4-x\sin xy}$ e. $\frac{5x^4-x\sin xy}{5y^4}$

26. Find the horizontal tangents of the curve $y=x^4-2x^2+1$

a. x=1,-1 b. x=0, 1 c. x=0, 1, -1 d. x=0 e. x=0,-1

27. Find $\frac{dy}{dx}$ if $y=(1-5x^2)(9x^2-180)$

a. $-180x^3+1818$ b. $-180x^3+1818x$ c. $45x^3+909x$ d. $-180x^4+1818x^2$ e. $-45x^3+909$

28. Find $\frac{dy}{dx}$ if $y=\frac{7x-8}{8x^2+3}$

a. $\frac{-56x^2+107x+45}{(8x^2+3)^2}$ b. $\frac{-56x^2+128x+21}{(8x^2+3)^2}$ c. $\frac{56x^3-112x^2+149x}{(8x^2+3)^2}$ d. $\frac{168x^2-128x+21}{(8x^2+3)^2}$ e. none of these

29. For the function $f(x)=3x^2$, at the point (4,48), find: i) the slope of the curve ii) the equation of the tangent line iii) the equation of the normal line.

a. $m=24$, $y=24x-48$, $y=24x+48$

b. $m=24$, $y=24x-48$, $y=-\frac{1}{24}x+48\frac{1}{6}$

c. $m=-\frac{1}{24}$, $y=-\frac{1}{24}x+48\frac{1}{6}$, $y=24x-48$

d. $m=-\frac{1}{24}$, $y=-\frac{1}{24}x-\frac{1}{48}$, $y=24x+48$

e. none of these

30. An equation of the line tangent to the graph of $y=x^3+3x^2+2$ at its point of inflection is…

a. $y=-3x+1$ b. $y=-3x-7$ c. $y=x+5$ d. $y=3x+1$ e. $y=3x+7$

31. What is $\lim_{x\to\infty}\left(\frac{\sqrt{9x^2+2}}{4x+3}\right)$?

a. $\frac{3}{2}$ b. $\frac{3}{4}$ c. $\frac{\sqrt{2}}{3}$ d. 1 e. The limit does not exist.

32. $\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)$?

a. 0 b. $\frac{1}{2}$ c. 1 d. $\frac{3}{2}$ e. The limit does not exist.

33. If $y = \cos^2 x - \sin^2 x$, then y' =

a. -1 b. 0 c. $-2(\cos x + \sin x)$ d. $2(\cos x + \sin x)$ e. $-4(\cos x)(\sin x)$

34. The graph of the function $f(x) = 2x^{5/3} - 5x^{2/3}$ is increasing on which of the following intervals.

I. $1 < x$ II. $0 < x < 1$ III. $x < 0$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

35. If $y = \cos^2(2x)$, then $\frac{dy}{dx} =$

(A) $2\cos 2x \sin 2x$
(B) $-4\sin 2x \cos 2x$
(C) $2\cos 2x$
(D) $-2\cos 2x$
(E) $4\cos 2x$

36. Of the following, the limit which exists is

(A) $\lim_{x\to 0}\left(\frac{1}{x^2}\right)$ (B) $\lim_{x\to 0}\left(\frac{|x|}{x}\right)$ (C) $\lim_{x\to 0}\left(\frac{1}{\sin x}\right)$

(D) $\lim_{x\to 0}(\ln x)$ (E) $\lim_{x\to 0}\left(\frac{x^2+2x}{x}\right)$

37. Which of the following functions is both continuous and differentiable at all x in the interval $-2 \le x \le 2$?

(A) $f(x) = |x^2 - 1|$
(B) $f(x) = \sqrt{x^2 - 1}$
(C) $f(x) = \sqrt{x^2 + 1}$
(D) $f(x) = \frac{1}{x^2 - 1}$

38. Find the point on the graph of $y = \sqrt{x}$ between (1, 1) and (9, 3) at which the tangent to the graph has the same slope as the line through (1, 1) and (9, 3).

(A) (1, 1)
(B) $(2, \sqrt{2})$
(C) $(3, \sqrt{3})$
(D) (4, 2)
(E) none of the above

39. Consider the function $f(x) = \frac{x^4}{2} - \frac{x^5}{10}$. The *derivative* of f attains its maximum value at $x =$

(A) 3
(B) 4
(C) 5
(D) 0
(E) there is no maximum

40. $\lim_{x \to 1} \left(\frac{\sqrt{x+3} - 2}{1 - x} \right)$

(A) 0.5
(B) 0.25
(C) 0
(D) -0.25
(E) -0.5

41. Let f be defined by $f(x) = \begin{cases} \frac{x^2 - 2x + 1}{x - 1} & \text{for } x \neq 1 \\ k & \text{for } x = 1. \end{cases}$

Determine the value of k for which f is continuous for all real x.

(A) 0
(B) 1
(C) 2
(D) 3
(E) none of the above

42. Let $f(x) = x^4 + ax^2 + b$. The graph of f has a relative maximum at $(0, 1)$ and an inflection point when $x = 1$. The values of a and b are

(A) $a = 1, \quad b = -6$
(B) $a = 1, \quad b = 6$
(C) $a = -6, \quad b = 5$
(D) $a = -6, \quad b = 1$
(E) $a = 6, \quad b = 1$

43. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

(A) 5 in^3/sec
(B) 10 in^3/sec
(C) 15 in^3/sec
(D) 20 in^3/sec
(E) 25 in^3/sec

44.

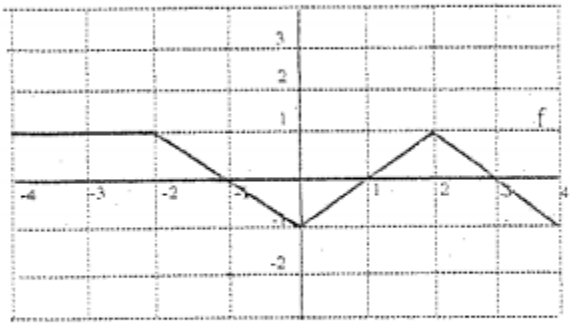
The graph of f is shown at the right. Which of the following statements are true?

I. $f(2) > f'(1)$

II. $\int_0^1 f(x)\,dx > f'(3.5)$

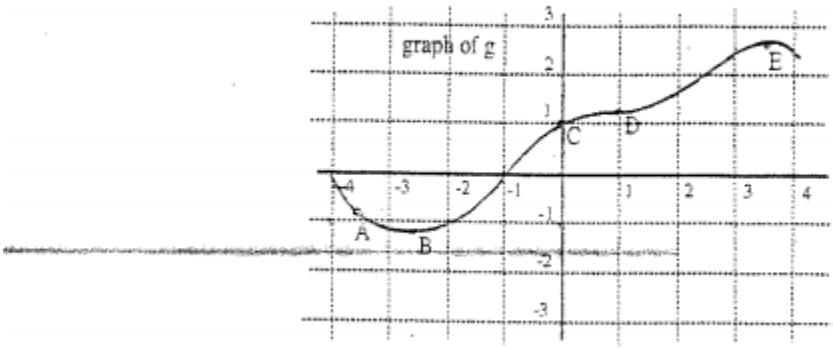
III. $\int_{-1}^{1} f(x)\,dx > \int_{-1}^{2} f(x)\,dx$

the graph of f



(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

45. At which point on the graph of $y = g(x)$ below is $g'(x) = 0$ and $g''(x) = 0$?



(A) A (B) B (C) C (D) D (E) E

46. If y is a differentiable function of x, then the slope of the tangent to the curve $xy - 2y + 4y^2 = 6$ at the point where $y = 1$ is

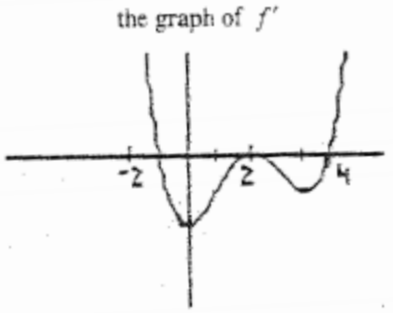
(A) $\frac{1}{12}$ (B) $-\frac{1}{10}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $-\frac{5}{6}$

CALCULATOR-ACTIVE

47. A farmer has 160 feet of fencing to enclose 2 adjacent rectangular pigpens. What dimensions should be used for each pigpen so that the enclosed area will be maximized?

A. None of these B. 80/3 ft x 20 ft C. 80/3 ft x 40ft D. 40 ft x 40 ft E. $4\sqrt{15}$ ft x $\frac{8}{5}\sqrt{15}$ ft

48. Let f be a function that has domain $[-2, 5]$. The graph of f' is shown at the right. Which of the following statements are TRUE?



I. f has a relative maximum at $x = -1$.
II. f has an absolute minimum at $x = 0$.
III. f is concave down for $-2 < x < 0$.
IV. f has inflection points at $x = 0$ and $x = 2$ and $x = 3$.

(A) I, II, IV (B) I, III, IV (C) II, III, IV (D) I, II, III (E) I, II, III, IV

49. On which interval is the graph of $f(x) = 4x^{3/2} - 3x^2$ both concave down and increasing?

(A) $(0, 1)$
(B) $\left(0, \frac{1}{2}\right)$
(C) $\left(0, \frac{1}{4}\right)$
(D) $\left(\frac{1}{4}, \frac{1}{2}\right)$
(E) $\left(\frac{1}{4}, 1\right)$

50. If $xy^2 = 20$ and x is decreasing at the rate of 3 units per second, the rate at which y is changing when $y = 2$ is nearest to

(A) -0.6 units/sec
(B) -0.2 units/sec
(C) 0.2 units/sec
(D) 0.6 units/sec
(E) 1.0 units/sec

51. The function f is defined on all the reals such that $f(x) = \begin{cases} x^2 + kx - 3 & \text{for } x \le 1 \\ 3x + b & \text{for } x > 1. \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(A) $k = -1, b = -3$
(B) $k = 1, b = 3$
(C) $k = 1, b = 4$
(D) $k = 1, b = -4$
(E) $k = -1, b = 6$

52. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

53. If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{-6}{7y^3}$ (B) $\frac{-3}{y^3}$ (C) 3 (D) $\frac{3}{2y}$ (E) $\frac{-9}{4y^3}$

54. The number of inflection points for the graph of $y = 2x + \cos(x^2)$ in the interval $0 \le x \le 5$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

55. A particle's position is given by $s(t) = 4t^3 - 9t^2 + 6t + 2$ for $t \ge 0$. When is the particle not moving?

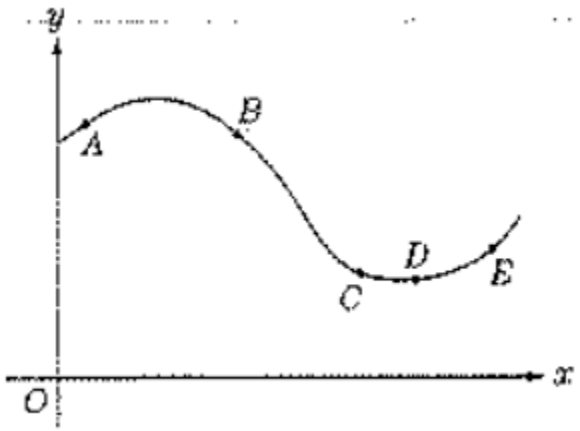
a. 3/2, ½ b. 3/2, 1 c. 1, ½ d. 1, 2 e. ½, 2

56. A particle moves along the x-axis so that at time $t \ge 0$, its position is given by $x(t) = (t + 1)(t - 3)^3$. For what values of t is the velocity of the particle increasing?

(A) all t (B) $0 < t < 1$ C) $0 < t < 3$ (D) $1 < t < 3$ E) $t < 1$ or $t > 3$

57. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

(A) A
(B) B
(C) C
(D) D
(E) E



58. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

(A) $-\frac{3}{2}$

(B) -1

(C) $-\frac{5}{14}$

(D) $-\frac{3}{14}$

(E) 0

59. If $f(x) = \sin^2(3 - x)$, then $f''(0) =$

(A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

60. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

61. Sand is falling into a conical pile at a rate of 10 cubic meters per second such that the height of the pile is always half of the diameter of the base of the pile. Find the rate at which the height of the pile is changing when the pile is 5 m high. (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$)

a. $\frac{1}{25\pi} m/s$ b. $\frac{2}{5\pi} m/s$ c. $\frac{4}{5\pi} m/s$ d. $\frac{8}{5\pi} m/s$ e. $250\pi m/s$