AP Calculus AB
Optimization – Day 1
HW 3-16

Name: Key

1) Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

sub $\rightarrow$ $xy = 192$

$x = \dfrac{192}{y}$

$x + 3y = M$ $\leftarrow$ min

$\dfrac{192}{y} + 3y = M$

$192y^{-1} + 3y = M$

$-192y^{-2} + 3 = 0$

$\dfrac{-192}{y^2} = -3$

$-192 = -3y^2$

$64 = y^2$ $\quad y = \pm 8$

$\boxed{y = 8}$

$x = \dfrac{192}{8}$

$\boxed{x = 24}$

2) A rectangle is bounded by the x-axis and the semicircle y = $\sqrt{16 - x^2}$. Find the dimensions of the rectangle that would maximize the area of the rectangle.

max $\rightarrow$ $A = 2xy$

$A = (2x)(\sqrt{16-x^2})$

$0 = (2)(\sqrt{16-x^2}) + (2x)(-2x)(\tfrac{1}{2})(16-x^2)^{-1/2}$

$0 = 2\sqrt{16-x^2} - \dfrac{2x^2}{\sqrt{16-x^2}}$

$\dfrac{2x^2}{\sqrt{16-x^2}} = \dfrac{2\sqrt{16-x^2}}{1}$

$2x^2 = 2(16-x^2)$

$2x^2 = 32 - 2x^2$

$4x^2 = 32$

$x^2 = 8$ $\quad x = \sqrt{8} = 2\sqrt{2}$

$y = \sqrt{16-(2\sqrt{2})^2}$ $\leftarrow$ sub

$y = \sqrt{16-8} = \sqrt{8} = \boxed{2\sqrt{2}}$

$\boxed{\begin{array}{l}\text{length} = 4\sqrt{2} \\ \text{width} = 2\sqrt{2}\end{array}}$

3) Find the point on the graph of $y = x^3$ that is closest to the point (4, 0).

$d = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(x-4)^2 + y^2} = \sqrt{(x^2-8x+16) + (x^3)^2}$

$d = \sqrt{x^2 - 8x + 16 + x^6}$

$0 = (2x - 8 + 6x^5)(\tfrac{1}{2})(x^2 - 8x + 16 + x^6)^{-1/2}$

$0 = \dfrac{x - 4 + 3x^5}{\sqrt{x^2 - 8x + 16 + x^6}}$

$0 = x - 4 + 3x^5$

use calculator! $\boxed{x = 1}$ $\quad \boxed{y = 1^3 = 1}$ $\quad \boxed{(1, 1)}$

4) A gardener wants to make a rectangular enclosure using a wall as one side and 120 m of fencing for the other three sides. Find the dimensions of the garden that will maximize the area of the garden.

sub

$2w+l=120$

$l=120-2w$

$lw=A \leftarrow$ max

$(120-2w)(w)=A$

$120w-2w^2=A$

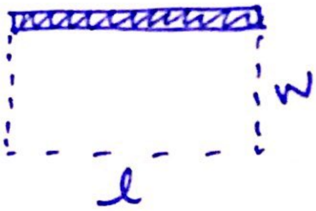
$120-4w=0$

$120=4w$

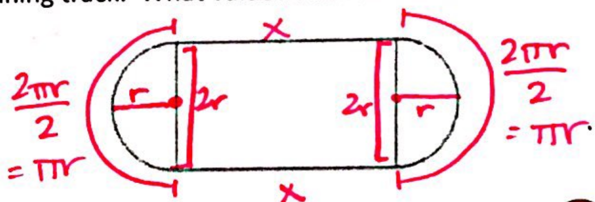
$\boxed{w=30m}$

$2(30)+l=120$

$\boxed{l=60m}$



5) An athletic field is to be built in the shape of a rectangle x units long with a semicircle of radius r at both ends. The field is to be bounded by a 400-m running track. What values of x and r will give the rectangle the largest possible area?



$A=2rx$

$400=2x+2\pi r$

$400-2\pi r=2x$

$200-\pi r=x$

max

$A=2r(200-\pi r)$

$A=400r-2\pi r^2$

$0=400-4\pi r$

$4\pi r=400$

$r=\frac{400}{4\pi}$

$\boxed{r=\frac{100}{\pi}m}$

$200-\pi\left(\frac{100}{\pi}\right)=x$

$200-100=x$

$\boxed{100m=x}$

6) A standard US can of soda holds 12 fluid ounces or 355 ml. Find the dimensions of a cylindrical can that will use the least amount of aluminum. (Remember to use the formulas for surface area and volume of a cylinder.)

$S=2\pi r^2+2\pi rh$

$V=\pi r^2h$

$355=\pi r^2h$ ← sub

$\frac{355}{\pi r^2}=h$

min

$S=2\pi r^2+2\pi r\left(\frac{355}{\pi r^2}\right)$

$S=2\pi r^2+\frac{710\cancel{\pi r}}{\cancel{\pi r^2}}$

$S=2\pi r^2+\frac{710}{r}$

$S=2\pi r^2+710r^{-1}$

$0=4\pi r-710r^{-2}$

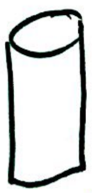
$0=4\pi r-\frac{710}{r^2}$

$\frac{710}{r^2}=4\pi r$

$710=4\pi r^3$

$\frac{710}{4\pi}=r^3$

$r=\sqrt[3]{\frac{710}{4\pi}}=3.8372\text{ cm}$



$355=\pi(3.8372)^2h$

$113=(3.8372)^2h$

$\boxed{h=7.6745\text{ cm}}$