AP Calculus AB
Related Rates – Cones, Spheres, and Triangular Prisms
HW 3-14

Name: Key

1) Water flows at the rate of 2 ft³/min into a tank in the shape of an inverted right circular cone of height 6 ft and radius 2 ft. At what rate is the surface of the water rising when the tank is half full? (Hint: Find the full volume first!)

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (4)(6) = 8\pi$

half full $\rightarrow 4\pi$

$\frac{dV}{dt} = 2$

$h = 6$

$r = 2$

$\frac{dh}{dt} = ?$

$V = 4\pi$

$\frac{1}{3}h = r$

$h = 3r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{9}h^2\right)h = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{3}{27}h^2\frac{dh}{dt}$

$2 = \frac{3}{27}(36)\frac{dh}{dt}$

$2 = \frac{1}{9}(36)\frac{dh}{dt}$

$2 = 4\frac{dh}{dt}$

$\boxed{\frac{1}{2}\text{ ft/min} = \frac{dh}{dt}}$

2) As sand leaks out of a hole in a container, it forms a conical pile whose altitude is always the same as its radius. If the height of the pile is increasing at a rate of 6 in/min, find the rate at which the sand is leaking out when the altitude is 10 inches.

$h = r$

$\frac{dh}{dt} = 6$

$\frac{dV}{dt} = ?$

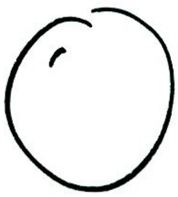
$h = 10$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^2 h = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi(10^2)(6) = \boxed{600\pi\text{ in}^3/\text{min}}$

3) Gas is escaping from a spherical balloon at a rate of 10 ft³/hr. At what rate is the radius changing when the volume is 400 ft³?



$\frac{dV}{dt} = -10$

$\frac{dr}{dt} = ?$

$V = 400$

Solve for r!

$400 = \frac{4}{3}\pi r^3$

$300 = \pi r^3$

$95.4930 = r^3 \quad r = 4.5708$

$V = \frac{4}{3}\pi r^3$

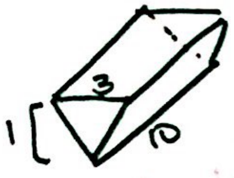
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-10 = 4\pi(4.5708)^2\frac{dr}{dt}$

$-10 = 262.5372\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = -.0381\text{ ft/hr.}}$

4) A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across the top and have a height of 1 ft. If the trough is being filled with water at the rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?



$V = \frac{1}{2}bhl = \frac{1}{2}(3h)hl = \left(\frac{3}{2}h^2 l\right)$

$\frac{1}{3}b = h \rightarrow b = 3h$

$\frac{dV}{dt} = \left(3h\frac{dh}{dt}\right)(l) + \left(\frac{3}{2}h^2\right)\left(\frac{dl}{dt}\right)$

$b = 3$, $h = 1$, $l = 10$

$12 = 3\left(\frac{1}{2}\right)\left(\frac{dh}{dt}\right)(10) + \left(\frac{3}{2}\left(\frac{1}{2}\right)^2\right)(0)$

$\frac{dV}{dt} = 12$

$12 = \frac{15}{\cancel{30}}\frac{dh}{dt}$

$\frac{dh}{dt} = ?$

$\frac{12}{15}\ \cancel{\frac{12}{30}} = \frac{dh}{dt}$

$\boxed{\frac{4}{5}\ \text{ft/min} = \frac{dh}{dt}}$

$h = \cancel{6}\ 0.5 = \frac{1}{2}$

5) Suppose a spherical snowball is melting and the radius is decreasing at a constant rate, changing from 12 inches to 8 inches in 45 minutes. How fast was the volume changing when the radius was 10 inches.

loses 4 in. in 45 min.

$\frac{4}{45} = \frac{?}{60}$

$240 = ? \cdot 45$

$? = \frac{16}{3}$

loses $\frac{16}{3}$ in in 1 hour.

$\frac{dr}{dt} = -\frac{16}{3}$

$\frac{dV}{dt} = ?$

$r = 10$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

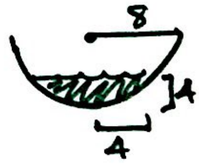
$\frac{dV}{dt} = 4\pi(100)\left(-\frac{16}{3}\right)$

$\boxed{\frac{dV}{dt} = -\frac{6400}{3}\pi\ \text{in}^3/\text{hr.}}$

6) Water is dripping into a hemispherical bowl with a radius of 8 cm at a rate of 1 cubic cm per minute. At what rate is the depth increasing when it is 4 cm?

* for spheres and hemispheres, the radius is always the same as the height.

$V = \frac{2}{3}\pi r^3$



$\frac{dV}{dt} = 1$

$\frac{dh}{dt} = ?$

$h = 4$

$r = 4$

$h = r$

$V = \frac{2}{3}\pi h^3$

$\frac{dV}{dt} = 2\pi h^2 \frac{dh}{dt}$

$1 = 2\pi(16)\frac{dh}{dt}$

$1 = 32\pi \frac{dh}{dt}$

$\boxed{\frac{1}{32\pi}\ \text{cm/min} = \frac{dh}{dt}}$