AP Calculus AB
Related Rates
HW 3-12

Name: Key

1) A fire has started in a dry, open field and spreads in the form of a circle. The radius of the circle increases at the rate of 6 ft/min. Find the rate at which the fire area is increasing when the radius is 150 ft.

$A = \pi r^2$, $\frac{dr}{dt} = 6$, $\frac{dA}{dt} = ?$, $r = 150$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$
$$\frac{dA}{dt} = 2\pi(150)(6)$$
$$\boxed{\frac{dA}{dt} = 1800\pi \text{ ft}^2/\text{min.}}$$

2) A cylinder filled with water has a 3 foot radius and 10 foot height. It is drained such that the depth of the water is decreasing at 0.1 feet per second. How fast is the water draining from the tank?

$V = \pi r^2 h$, $r = 3$, $h = 10$, $\frac{dh}{dt} = -0.1$, $\frac{dV}{dt} = ?$

$$\frac{dV}{dt} = \left(2\pi r \frac{dr}{dt}\right)(h) + (\pi r^2)\left(\frac{dh}{dt}\right)$$
$$\frac{dV}{dt} = 2\pi(3)(0)(10) + \pi(3^2)(-0.1)$$
$$\boxed{\frac{dV}{dt} = -0.9\pi \text{ ft}^3/\text{sec}}$$

3) A stone is dropped into a lake, causing circular waves whose radii increase at a constant rate of 0.5 m/sec. At what rate is the circumference of a wave changing when its radius is 4 meters?

$C = 2\pi r$, $\frac{dr}{dt} = 0.5$, $\frac{dc}{dt} = ?$, $r = 4$

$$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$$
$$\frac{dc}{dt} = 2\pi(0.5)$$
$$\boxed{\frac{dc}{dt} = \pi \text{ m/sec}}$$

4) Boyle's law states that when a sample of gas is compressed at a constant temperature P and volume V are related by the equation PV=C, where C is a constant. Suppose that at a certain instant the volume is 600 cm^3 and the pressure is 150 kg/cm and is increasing at a rate of 10 kg/cm^2 every minute. At what rate is the volume decreasing at this instant?

$PV = C$, $V = 600$, $P = 150$, $\frac{dP}{dt} = 10$, $\frac{dV}{dt} = ?$

$$\frac{dP}{dt}(V) + P\left(\frac{dV}{dt}\right) = 0$$
$$10(600) + 150\frac{dV}{dt} = 0$$
$$6000 + 150\frac{dV}{dt} = 0$$
$$150\frac{dV}{dt} = -6000$$
$$\boxed{\frac{dV}{dt} = -40 \text{ cm}^3/\text{min}}$$

5) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

$\frac{dr}{dt} = 4$

$\frac{dA}{dt} = ?$

$r = 5$

$A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(5)(4) = \boxed{40\pi \text{ cm}^2/\text{min}}$$

6) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 9π m²/min. How fast is the radius of the spill increasing when the radius is 10 m?

$A = \pi r^2$

$\frac{dA}{dt} = 9\pi$

$\frac{dr}{dt} = ?$

$r = 10$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$9\pi = 2\pi(10)\left(\frac{dr}{dt}\right)$$

$$9\pi = 20\pi \frac{dr}{dt}$$

$$\frac{9\cancel{\pi}}{20\cancel{\pi}} = \frac{dr}{dt}$$

$$\boxed{\frac{9}{20} \text{ m/min.} = \frac{dr}{dt}}$$