AP Calculus AB

Linearization

HW 3-9

Name: Key

Use linearization to complete the approximation, and state if it is an overestimation or an underestimation.

1) $f(x) = x^3 - 2x + 3$, approximate $f(2.1)$

easy: $x = 2$

$f(2) = 2^3 - 2(2) + 3 = 7$

point: $(2, 7)$

$f'(x) = 3x^2 - 2$

$f'(2) = 10$

$y - 7 = 10(x - 2)$

$y - 7 = 10x - 20$

$y = 10x - 13$

$10(2.1) - 13$

$= 21 - 13 = \boxed{8}$

$f''(x) = 6x$

$f''(2) = 12$

concave up

underestimation

2) $f(x) = \sqrt{x^2+9}$, $= (x^2+9)^{1/2}$ approximate $f(-3.9)$.

easy: $x = -4$

$f(-4) = \sqrt{(-4)^2 + 9} = \sqrt{25} = 5$

point: $(-4, 5)$

overestimate

$f'(x) = (2x)\left(\frac{1}{2}\right)(x^2+9)^{-1/2} = \frac{x}{\sqrt{x^2+9}}$

$f'(-4) = \frac{-4}{\sqrt{25}} = -\frac{4}{5} = m$

$y - 5 = -\frac{4}{5}(x + 4)$

$y - 5 = -\frac{4}{5}x - \frac{16}{5}$

$y = -\frac{4}{5}x - \frac{16}{5} + \frac{25}{5}$

$y = -\frac{4}{5}x + \frac{9}{5}$

$-\frac{4}{5}\left(\frac{-39}{10}\right) + \frac{9}{5}$

$= \frac{156}{50} + \frac{9}{5}$

$= \frac{156}{50} + \frac{90}{50} = \frac{246}{50}$

$\frac{123}{25}$

$f''(x) = \frac{\sqrt{x^2+9} - \frac{x^2}{\sqrt{x^2+9}}}{x^2+9} = -$

3) Approximate $\sqrt{101}$.

$y = \sqrt{x}$

easy: $x = 100$

$f(100) = \sqrt{100} = 10$

point: $(100, 10)$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20} = m$

$y - 10 = \frac{1}{20}(x - 100)$

$y - 10 = \frac{1}{20}x - 5$

$y = \frac{1}{20}x + 5$

$y = \frac{1}{20}(101) + 5 = \frac{101}{20} + \frac{100}{20} = \boxed{\frac{201}{20}}$

$f''(x) = -\frac{1}{4}x^{-3/2}$

$= \frac{-1}{4x^{3/2}} = \frac{-1}{4\sqrt{x^3}}$

$f''(100) = \frac{-1}{4\sqrt{100^3}} = -$

concave down

overestimate

4) Approximate $\cos 62°$.

easy: $x = 60$

$f(x) = \cos x$

$f(60) = \cos(60) = \frac{1}{2}$

point: $(60, \frac{1}{2})$

$f'(x) = -\sin x$

$f'(60) = -\sin 60 = -\frac{\sqrt{3}}{2} = m$

$f''(x) = -\cos x$

$-\cos(60) = -\frac{1}{2}$

c. down

overestimate

$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}(x - 60)$

$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}x + \frac{60\sqrt{3}}{2}$

$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}x + \frac{60\sqrt{3}}{2}$

$y = -\frac{\sqrt{3}}{2}x + \frac{60\sqrt{3}}{2} + \frac{1}{2}$

$-\frac{\sqrt{3}+1}{2}$

$y = \frac{62\sqrt{3}}{2} + \frac{60\sqrt{3}}{2} + \frac{1}{2} = \frac{-2\sqrt{3}}{2} + \frac{1}{2}$