AP Calculus AB

HW 3-8

AB Calculus Information About f(x) given f'(x) Homework

Name: Key

1. Complete each of the following statements.

a) When f' is positive, the graph of f is increasing.

b) When f' is negative, the graph of f is decreasing.

c) When f'' is positive, the graph of f is concave upward.

d) When f'' is negative, the graph of f is concave downward.

e) When f' is increasing, the graph of f is concave upward.

f) When f' is decreasing, the graph of f is concave downward.

x	f(x)
-1	3
1	7

2. Use the function $f(x) = 3x - x^3 + 5$ to answer the following.

Increasing: $(-1,1)$ because $f'(x)>0$

Decreasing: $(-\infty,-1) \cup (1,\infty)$ because $f'(x)<0$

a) Find the intervals that the function is increasing. Justify your response.

$f'(x) = 3 - 3x^2$

$3 - 3x^2 = 0$

$3(1-x^2) = 0$

$3(1+x)(1-x) = 0$

$1+x=0$ | $1-x=0$

$\boxed{x=-1}$ | $\boxed{x=1}$

$f'(-2) = 3-3(-2)^2 = -9$

$f'(0) = 3-3(0)^2 = 3$

$f'(2) = 3-3(2)^2$
$= -9$

b) Find the intervals that the function is decreasing. Justify your response.

See above.

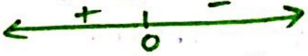
c) Find the intervals that the function is concave up. Justify your response.

$f''(x) = 0 - 6x$

$f''(x) = -6x$

$-6x = 0$

$x = 0$



$f''(-1) = 0-6(-1) = 6$

$f''(1) = 0-6(1) = -6$

c.up: $(-\infty, 0)$ because $f''(x)>0$

c.down $(0,\infty)$ because $f''(x)<0$

d) Find the intervals that the function is concave down. Justify your response.

See above.

e) Find the x-coordinates of all points of inflection for the function. Justify your response.

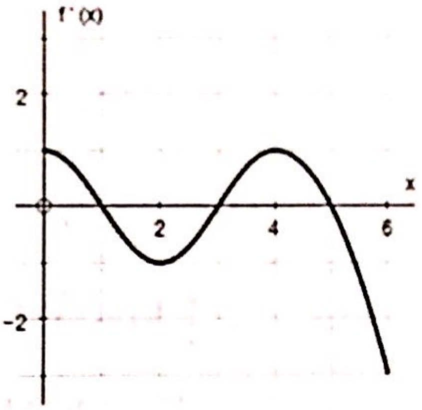
$x=0$, because the concavity changes from concave up to concave down.

f) Find all relative extrema and label them as a maximum or minimum. Justify your response.

Local minimum at $(-1, 3)$ because $f(x)$ goes from dec. to inc.

Local maximum at $(1, 7)$ because $f(x)$ goes from inc. to dec.

3. Use the graph of $f'(x)$ defined over the interval $[0, 6]$ provided below to answer the following.



a) When is $f(x)$ increasing? When is $f(x)$ decreasing? Justify your responses.

Inc: $(0,1) \cup (3,5)$ b/c $f'(x) > 0$.

Dec: $(1,3) \cup (5,6)$ b/c $f'(x) < 0$.

b) Determine the x-coordinates of all local extrema. Justify your response.

Local max: $x=1, x=5$ because $f'(x)$ goes from positive to negative

Local min: $x=3$ because $f'(x)$ goes from negative to positive

c) When is f concave up? When is f concave down? Justify your responses.

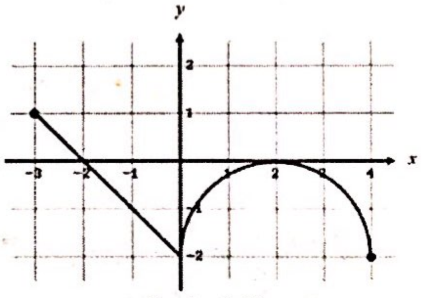
C. up: $(2,4)$ because $f'(x)$ is increasing

C. down: $(0,2) \cup (4,6)$ because $f'(x)$ is decreasing

d) Find the x-coordinates of all points of inflection. Justify your response.

$x=2$, $x=4$ because $f'(x)$ goes from increasing to decreasing or vice versa.

4. Let f be a function defined on the closed interval $[-3, 4]$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Graph of f'

a) When is $f(x)$ increasing? When is $f(x)$ decreasing? Justify your responses.

Inc: $(-3,-2)$ because $f'(x) > 0$

Dec: $(-2,4)$ because $f'(x) < 0$

b) Determine the x-coordinates where $f(x)$ has a relative maximum. Justify your response.

$x=-2$, because $f'(x)$ goes from positive to negative.

c) When is f concave up? When is f concave down? What are the x-coordinates of the points of inflection? Justify your responses.

concave down: $(-3,0) \cup (2,4)$ because $f'(x)$ is decreasing

concave up: $(0,2)$ because $f'(x)$ is increasing

d) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

$m=-2$

$y-3=-2(x-0)$

$y-3=-2x$

$\boxed{y=-2x+3}$