

1) Determine whether $f(x)$ is continuous at the given point.

$$\text{a) } f(x) = \begin{cases} x^3 + 3x, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

at $x=0$

$$\text{b) } f(x) = \begin{cases} \frac{x^2 - 6x}{x^2 + 6x}, & x \neq 0 \\ -2, & x = 0 \end{cases}$$

at $x=0$

$$\text{c) } f(x) = \begin{cases} \frac{x^3 - 1}{x^2 - 1}, & x < 1 \\ 2, & x = 1 \\ \frac{3}{x+1}, & x > 1 \end{cases}$$

at $x=1$

2) Let g be a continuous function on the closed interval $[0, 1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is not necessarily true? Completely explain your answer.

A) There exists a number h in $[0, 1]$ such that $g(h) \geq g(x)$ for all x in $[0, 1]$.

B) For all a and b in $[0, 1]$, if $a = b$, then $g(a) = g(b)$.

C) There exists a number h in $[0, 1]$ such that $g(h) = \frac{1}{2}$.

D) There exists a number h in $[0, 1]$ such that $g(h) = \frac{3}{2}$.

E) For all h in the open interval $(0, 1)$, $\lim_{x \rightarrow h} g(x) = g(h)$.

3) If $\lim_{x \rightarrow a} f(x) = L$ and L is a real number, which of the following must be true? Completely explain your answer.

A) $f(x)$ is continuous at $x=a$.

B) $f(x)$ is defined at $x=a$.

C) $f(a) = L$

D) None of the above
