More Compound Interest . . . Making even more money by investing

In the previous lesson, we explored compound interest investments where the investments were compounded annually. Many banks and credit unions offer investments which are compounded semi-annually, quarterly, monthly, or daily in order to earn even more money. We will explore some of those investments in this lesson.

Mr. Watson sold his boat for $10,000. He wants to invest the money. How much money will he have after 1 year if he invests the $10,000 in an account that pays 4% compounded interest per year?

That's pretty easy now. The initial investment is $10,000, the interest rate is 4%, and the time is 1 year. Using the formula we found in the previous lesson, $y = a \bullet (1 + r)^t$, he will have:

$Y = 10{,}000(1 + 0.04)^1$

$Y = 10{,}000(1.04)^1$

$Y = 10{,}400$ at the end of 1 year.

Mr. Watson sees an advertisement for another type of savings account:

4% interest per year compounded quarterly

Mr. Watson doesn't really understand what "quarterly" means in terms of investing, so he asks the bank officer to explain this to him. The teller explains that instead of giving him 4% of 10,000 at the end of one year, the bank will give him 1% of 10,000 at the end of each 3-month time period, which is one quarter of the year. Here's how this breaks down:

The initial investment is $10,000 at 0 months of the year.

In 3 months, the investment is 1% of 10,000, which is 10,000 + 0.01(10,000) = 10,000 + 100 = 10,100

In 6 months, the investment is 1% of 10,100, which is 10,100 + 0.01(10,100) = 10,100 + 101 = 10, 201

In 9 months, the investment is 1% of 10,201, which is 10,210 + 0.01(10,201) = 10,201 + 102.01 = 10,303.01

In 12 months (1 year), the investment is 1% of 10,303.01, which is 10,303.01 + 0.01(10,303.01) = 10,303.01 + 103.03 = 10,406.04

So, Mr. Watson will be able to earn a little more money ($6.04) by compounding quarterly.

Take a few minutes to write the NOW-NEXT and explicit equations of the compounding quarterly situation.

Let's see . . . the initial amount is $10,000, the interest rate is 4%, and the time is 1 year . . . but the investment was calculated 4 times during the year at 1 % interest. So the interest rate was split into 4 equal parts (1%) . . . over the course of the year . . . but the investment was calculated 4 times during the year.

$\text{NEXT} = \text{NOW} \bullet (\text{interest rate/number of time compounded in the year})(\text{NOW})^{\text{time} \bullet \text{number of times compounded in the year}}$

$\text{NEXT} = \$10{,}000(1 + (4\%/4))^{\text{time} \bullet 4}$

$\text{NEXT} = \$10{,}000(1.01)^{1 \bullet 4} =$

$\text{NEXT} = \$10{,}406.04$

Okay, so let's put this into an explicit formula so that we can use it in a more efficient way.

$Y = \text{initial investment}(1 + \text{interest rate/number of time compounded})^{\text{time} \bullet \text{times compounded in the year}}$

$Y = a \bullet (1 + r/n)^{nt}$, where a = initial amount, r = interest rate, t = time, and n = number of times the investment is compounded during one year. This is the formula that I want you to remember.

Back to the Story

Mr. Watson sees an advertisement for a different bank that offers 4% interest compounded monthly, which means that he will get 1/12 of 4% interest every month. How much money will he have at the end of the year if he invests his money at this bank?

Let's try the explicit formula for this situation.

$Y = 10{,}000 \bullet (1 + .04/12)^{(1 \bullet 12)}$

$Y = 10{,}000(1 + 0.0033)^{12}$

$Y = 10{,}000(1.0033)^{12}$

$Y = 14{,}763.99$

This is a much better yield on the investment, even though it is just for one year. In this investment, Mr. Watson made $4,763.99 on his $10,000 investment. Woot!