Compound interest

Once in a while, you hear about people winning the lottery. The winnings can be several millions of dollars. The big money winners are usually paid in annual installments for about 20 years. But some of the smaller prizes are awarded in a matter of weeks. What do you think you would do if you won the lottery?

Sue's uncle gave her a lottery ticket on her 18th birthday and she won!! In the lottery payoff scheme, she has two payoff choices:

Option 1 is to receive a single $20,000 payment now.

Option 2 is to receive a single $40,000 payment in ten years.

Which option should she take?

But wait, the word is out and several banks have called to tell you about their investment plans for Option 1. One bank has offered a special 10-year certificate of deposit paying 8% interest compounded annually. Should Sue take option 1 and invest with this bank?

How do you represent and reason about functions involved in investments paying compound interest anyway?

It's not really that difficult. Let's look at each option above.

Option 2 is really easy; in 10 years from now, you'll receive 40,000.

Option 1 is not as clear. If you take option 1, you'll receive 20,000 and that is it. However, if you invest this money in the special 10 year certificate of deposit, you'll receive more. Let's find out how much more.

At the end of year 1, your balance will be: 20,000 + (0.08 x 20,000) = 20,000 + 1,600 = 21,600
At the end of year 2, your balance will be: 21,600 + (0.08 x 21,600) = 21,600 + 1,728 = 23,328
At the end of year 3, your balance will be: 23,328 + (0.08 x 23,328) = 23,328 + 1866.24 = 25,194.24

Wait a minute – there has to be an easier way to do this . . . let's write a NOW-NEXT equation.

To get the NEXT year's value, I take the NOW value and add it to 8% of it's value. So, the initial value is $20,000, the interest rate is 8% and the investment is compounded once a year for 10 years.
Here's what I know so far: $NEXT = NOW + 0.08NOW^{10}$ If I factor out the NOW . . . I get
$NEXT = NOW(1 + 0.08)^{10}$ If I add the 1 to 0.08 . . . I get
$NEXT = NOW\ (1.08)^{10}$

So, let's write this as an explicit function and add in the information that we have:

$$y = a * (1 + r)^t$$

where a = the initial amount (20,000), r = the interest rate in decimal form (0.08), and t = time in years (10)

$$y = 20{,}000(1 + 0.08)^{10}$$
$$y = 20{,}000(1.08)^{10}$$
$$y = 43{,}178.50$$

Which option should Sue take? We'll I would advise her to take option 1 and to invest the funds in the special CD. However, if she doesn't want to invest her money, then she would be better off, taking option 2. The down side here is that she will have to wait 10 years for her money. Which option would you advise Sue to take? Why?

In general, I want you to remember compound interest as the formula below:

$$y = a * (1 + r)^t$$

where a = the initial amount, r = the interest rate in decimal form, and t = time in years. Later we will look at investments where time is compounded more than once a year.

YOUR TURN

Write the NOW-NEXT and explicit formulas for the following compound interest problems.

1) You have an initial investment of $15,000 to be invested at a 6% interest rate compounded annually. What is the investment worth at the end of 5 years? What is the investment worth at the end of 15 years?

2) You have an initial investment of $7,000 to be invested at a 4.5% interest rate compounded annually. What is the investment worth at the end of 20 years? What is the investment worth at the end of 30 years?

3) Sam's aunt Matilda gave him a stamp collection worth $2,500. Sam is considering selling the collection, but his aunt told him that, if he saved it, the stamps would increase in value. Sam decided to save the collection, and it's value increased by 3.75% each year. Find the value of the collection 5 years from now. When will it be worth $5,000?